Computing Fifth Geometric-Arithmetic Index of Dendrimer Nanostars

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1. Introduction

Let \( G=(V; E) \) be a simple connected graph. In chemical graph theory, the sets of vertices and edges of \( G \) are denoted by \( V=V(G) \) and \( E=E(G) \), respectively. A molecular graph is a simple finite graph such that its vertices correspond to the atoms and the edges to the bonds. A general reference for the notation in graph theory is [1-3].

In chemical graph theory, we have many different topological index of arbitrary molecular graph \( G \). One of important connectivity topological indices is Geometric-Arithmetic (GA) index of \( G \) and was introduced by Vukicevic and Furtula [4, 5], in 2009 and defined as

\[
GA(G) = \sum_{uv \in E(G)} \frac{2\sqrt{d_u d_v}}{d_u + d_v}
\]

where \( d_u \) denotes the degree of vertex \( u \) of \( G \). In 2011, A. Graovac et al defined a new version of GA index as

\[
GA_s(G) = \sum_{uv \in E(G)} \frac{2\sqrt{S_v S_u}}{S_v + S_u}
\]

where \( S_u = \sum_{v \in N_u} d_v \) to be the summation of degrees of all neighbors of vertex \( u \) in \( G \) and \( N_u = \{ v \in V(G) \mid uv \in E(G) \} \). The reader may consult [7-19] and references cited therein.

In this paper, we give explicit formulas for the fifth geometric-arithmetic index of an infinite class of dendrimer nanostars. For further study on this topic, we encourage the reader to consult papers [20-29]. In this paper, for every infinite integer \( n \), \( D_s[n] \) denotes the \( n \)th growth of nanostar dendrimer. In following figures, a kind of 3rd growth of dendrimer and \( D_s[0] \) are shown. Here our notations are standard and mainly taken from standard books of chemical graph theory [1-3].
2. Main results and discussion

Let $D_3[n]$ denote a kind of dendrimer nanostars with $n$ growth stages, see for example Figures 1 and 2, the goal of this paper is to compute a closed formula of this new Connectivity index “fifth geometric-arithmetic index $GA_5$” of $D_3[n]$ for every $n \geq 0$ as follows:

**Theorem 1.** The fifth geometric-arithmetic index $GA_5$ of Nanostar Dendrimer $D_3[n]$ for every $n \geq 0$ is equal to

$$GA_5(D_3[n]) = 3(2^n - 2) + 6(2^n - 2) + 9(2^n - 2) + 12(2^n - 2) + 15(2^n - 2) - 8.$$  

**Proof.** Consider Nanostar Dendrimer $D_3[n]$ for every $n \geq 0$, (see Figures 1 and 2). From Figure 2 and Ref. [20], one can see that the number of vertices/atoms in this nanostar is equal to $|V(D_3[n])| = 24(2^n - 2)$ and also the number of edges/bonds is $|E(D_3[n])| = 24(2^n - 2) + 2$. Since all vertices/atoms of nanostar $D_3[n]$ have degree 3, 2 and 1 (hydrogen (H) atom), we divide the vertex/atom set of $D_3[n]$ in three partitions as

- $V_1 = \{v \in V(D_3[n]) | d_v = 3\}$
- $V_2 = \{v \in V(D_3[n]) | d_v = 2\}$
- $V_3 = \{v \in V(D_3[n]) | d_v = 1\}$

By according to the 2-Dimensional of dendrimer $D_3[n]$ in Figure 2, one can see that $|V_1(n)| = 3(2n)$ and $|V_2(n)| = 12(2n + 1 - 1)$. And also, summation of degrees of edge endpoints of this nanostar have six types $e(3,5)$, $e(5,5)$, $e'(5,5)$, $e(5,7)$, $e(7,9)$ and $e(9,9)$ that are shown in Figure 2 by red, yellow, green, blue, hoary and black colors. Since for all edge $e = uv$ of the types $e(3,5)$, $S_v = 3$ (for all hydrogen H atom) and $S_u = 5$, such that vertices $x$ and $y$ are one of adjacent vertices of degree 2 and other types are analogous. From Figure 2, the number of edges of these edge types are shown in following table.

<table>
<thead>
<tr>
<th>Summation of degrees of edge endpoints</th>
<th>$e_{(3,5)}$</th>
<th>$e_{(5,5)}$</th>
<th>$e'_{(5,5)}$</th>
<th>$e_{(5,7)}$</th>
<th>$e_{(7,9)}$</th>
<th>$e_{(9,9)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of edges of this type</td>
<td>$3(2^n)$</td>
<td>$6(2^n - 1)$</td>
<td>$9(2^n - 1)$</td>
<td>$15(2^n - 1)$</td>
<td>$25(2^n - 1)$</td>
<td>$35(2^n - 1)$</td>
</tr>
</tbody>
</table>

Thus, by using above table and Figure 3, we can deduce the following formula for fifth geometric-arithmetic index $GA_5$ of Nanostar Dendrimer $D_3[n]$ for $n \geq 0$, as follow:

$$GA_5(D_3[n]) = \sum_{uv \in E(D_3[n])} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$$

$$= \sum_{uv \in E_{(3,5)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v} + \sum_{uv \in E_{(5,5)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v} + \sum_{uv \in E_{(5,5)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v}$$

$$+ \sum_{uv \in E_{(5,7)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v} + \sum_{uv \in E_{(7,9)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v} + \sum_{uv \in E_{(9,9)}} \frac{2\sqrt{S_u S_v}}{S_u + S_v}.$$
Finally, the Fifth Geometric-Arithmetic index of $D_3[n]$ is equal to

$$GA_5(D_3[n]) = 2^9 \left(3^3 + 2^{n-3} \left(6\sqrt{5} + 9\sqrt{7} - 2\sqrt{35} + 18\right)\right).$$

Here, we complete the proof of the theorem.

References